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Model-based Tomographic Reconstruction Literature Search

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Summary

In the process of preparing a proposal for internal research funding, a literature search was conducted on the subject of model-based tomographic reconstruction (MBTR). The purpose of the search was to ensure that the proposed research would not replicate any previous work. We found that the overwhelming majority of work on MBTR which used parameterized models of the object was theoretical in nature. Only three researchers had applied the technique to actual data. In this note, we summarize the findings of the literature search.

In the literature, the term “model-based reconstruction” is used to describe several different approaches to tomographic reconstruction or inversion. For our purpose we use it to describe any method of reconstruction that uses a parametric model of the object instead of a voxel (pixel) representation. That is, if the object is a circle, we represent it as a center coordinate and radius (3 parameters) instead of sampling the plane uniformly at a given resolution and setting each pixel to zero or one based on whether the pixel is inside or outside the circle ($\sim N^2$ parameters, where N is the number of pixels along the diameter). The difference between the two ways of representing the object cannot be overemphasized when searching the literature. The term “model-based reconstruction” most often refers to a numerical model of the wave propagation through the object, not specifically to a model-based (parametric) representation of the object. Thus a simple search of the term “model-based reconstruction” can generate a large number of references that are not directly related to the proposed work. For example, of the first 200 references (out of 580) in a Google search of “model-based reconstruction” (<http://scholar.google.com/scholar?q=model-based+tomographic+reconstruction&ie=UTF-8&oe=UTF-8&hl=en&btnG=Search>), only 5 use parametric representations of the object.

For our literature search we used the Inspec and Web of Science databases. An initial list of references was obtained from Inspec by finding the intersection of searches for “model-based tomography” and “object”. The 41 references obtained from this search were further reduced to 11 by selecting only those that used parametric representations of the object. The Web of Science database was then used to follow the citations from these papers to obtain the final list of over 40 references.

The earliest reference to a parametric description of the object is the well-known work by Rossi and Willsky [1, 2, 3], who calculated a maximum likelihood (ML) estimate of location, size, and orientation of single circles and ellipses directly from projections of the object. They showed that the maximum likelihood approach was fairly robust to the presence of obscuring objects and object model errors. In addition, they were able to characterize the errors in the model parameters by calculating the Cramer-Rao lower bounds of the variances. Follow on work by Sauer and Liu [4], Wang, *et al.* [5], and Karl, *et al.* [6] generalized this to multiple ellipses and three-dimensional ellipsoids. Milanfor, *et al.* [7, 8] applied the maximum likelihood method to parametric models of polygons. The elliptical object model was extended to cylinders by Bresler [9, 10] and Fessler [11], who also used both minimum mean-square error (MMSE) and maximum *a posteriori* (MAP) approaches to estimating the object parameters. All of this work tried to estimate the model parameters (and Cramer-Rao lower bounds) directly from simulated projections through the object. However, none applied these techniques to actual x-ray data.

Motivated by the early work of Rossi [1], Devaney and associates applied maximum likelihood to estimating object location in diffraction tomography [12, 13, 14]. This was applied to actual ultrasound data by Tsihirintzis in 1993 [15]. Later Naidu and Buvanewari [16] extended the object model approach in diffraction tomography to estimating the size and orientation of elliptical objects.

In the case where there is a single object in a constant background, one can choose to parameterize only the perimeter of the object using B-splines, Fourier series, or wavelets. Ye, Bresler, and Moulin used this approach to estimate object shape [17, 18, 19] for both radar imaging and tomography. Mohammed-Djafari and Soussen reconstructed general polyhedral objects by estimating the vertices directly from projections using both ML and MAP approaches [20, 21, 22, 23]. Finally, Feng, *et al.* [24], used a level-set approach to estimating the contours of objects.

Independently of these efforts, Hanson and Cunningham at Los Alamos National Laboratory have developed a Bayesian approach (MAP estimate) for model-based tomography. This was first described in a paper by Hanson [25] where it was applied to a pixel-based representation of the object. Later, it was used to estimate points on a closed contour between two regions in two dimensions [26]. The approach was formalized in a software package, the Bayesian Inference Engine (BIE), by Cunningham in 1994 [27, 28, 29], and subsequently applied to three-dimensional objects and real data [30, 31, 32, 33, 34]. In these applications, only the interface between the object and external region is estimated. The interface is represented by a triangulated surface (or general polygon in two dimension) defined by a set of vertices. The BIE is used to estimate the vertices directly from the projection data. Particular emphasis is placed on the ability to reconstruct and track a moving interface from projection data [35, 36]. The accuracy of the BIE approach is difficult to assess since it does not provide a direct way of estimating the variances of the parameters. In 1997, Hanson, Cunningham, and McKee [37, 38] used Markov chain Monte Carlo to estimate the accuracy of the perimeter for a simple two-dimensional pixel-based object. Though computationally intensive, this is the only work that calculated the true variances of the object parameters instead of just lower bounds. Such an approach for calculating variances would be impractical for more than a few test objects.

There are a few additional references from groups not associated with those discussed previously that have some relevance to object models for tomography. There has been some work in biomedicine trying to determine the position of prostheses from single x-ray images [39]. Noble *et al.* [40] used coordinate transformations to fit a CAD model of a part to stereo x-ray images. This led to a device that was actually deployed on an assembly line. West and Williams [41, 42] estimated the interface between fluid and air at different levels of a hydrocyclone. This was modeled as a circle with a given center and radius (3 parameters). They used both electrical resistance tomography and ultrasound to determine the best values for the center and radius. This is also the only example where two different tomographic measurements were combined to determine the best overall parameter values (data fusion).

From this survey, we see that there has been a fair amount of theoretical work on extracting model parameters for simple objects directly from projections. However, only Noble [40] applied this technique to actual industrial parts using CAD representations. In addition, most of the work used simulated data, not actual data, where measurement models will be important for attaining good performance. No work was found that combined all the features one would like in a practical implementation of model-based tomography: parameteric object models, realistic wave propagation code, estimation of model parameter variances, and model adaptivity. Any reconstruction code that combined all these features would be a significant breakthrough in practical tomography and inversion.

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